Takeoff Performance of Jet-Propelled Conventional and Vectored-Thrust STOL Aircraft

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An analytical method for the determination of the air distance of jet-propelled conventional (CTOL) and vectored-thrust short takeoff and landing (STOL) aircraft has been developed. The method assumes constant lift and drag coefficients during the climb and a constant value of the horizontal acceleration based on the aircraft's average velocity from touchoff to the 35- or 50-ft obstacle. It is indicated by this method that some "classical" or specification methods for computing CTOL air distances are not generally applicable. Design charts are developed for determining the ground, air, and total takeoff distance in terms of: an STOL thrust-to-weight ratio, $(T_x/W)/(1 - T_y/W)$; an effective wing loading $(W/S)(1/\sigma C_{L_o})$; and the L/D ratio. A parametric study of approximately 100 hypothetical CTOL and STOL aircraft was made, and graphs of the takeoff performance presented. It is shown that thrust-to-weight ratios greater than approximately 0.6 are required in order to show performance gains by thrust vectoring, that the major improvement in takeoff distance derives from the reduction in ground roll while air distance is relatively unaffected, and that reduced wing loadings or improved high-lift capabilities are equally as beneficial.

Nomenclature

a = acceleration, ft/sec^2
C_D = airplane coefficient of drag, $C_D = D/Sq$
C_L = airplane coefficient of lift, $C_L = L/Sq$
D = airplane drag, aerodynamic force component parallel to V, lb
D_L = C_D/C_L
g = acceleration due to gravity, 32.2 ft/sec^2
h = height of obstacle to be cleared at termination of takeoff climb, ft
K_y, K_L = constants defined by Eqs. (23) and (24)
L = airplane lift, aerodynamic force component perpendicular to V, lb
q = dynamic pressure, $\frac{1}{2}\rho V^2$, psf
S = wing reference area, ft^2
t = time, sec
T = engine net thrust, lb
$\tilde{T}_w$ = $(T_x/W)/(1 - T_y/W)$
u = horizontal component of velocity, fps
v = vertical component of velocity, fps
V = flight-path velocity, fps or knots, as noted
V_g = potential velocity $V_g = (2gh)^{1/2}$
W = airplane gross weight, lb
$\tilde{W}_s$ = $(W/S)(1/\sigma C_{L_o})$, psf
x, X = horizontal distance, measured from initial position on runway to airplane c.g., ft
y, Y = vertical position of airplane c.g., above ground line, ft
Z = nondimensional acceleration parameter $(V_g/V_o) \times (a_x/g)_{av}(1 - T_{yo}/W)^{-1/2}$
α = angle of attack, rad or deg
γ = flight-path angle, rad or deg
θ = pitch attitude of airplane, rad or deg
Λ = thrust-line inclination relative to airplane waterline, deg
μ = rolling coefficient of friction
ρ = atmosphere air density (ρ_{SL} = 0.002378 slugs/ft^3)
σ = density ratio, $\sigma = \rho/\rho_{SL}$

Subscripts

a = air
av = average
c = climb
eff = effective
g = ground
$a + g$ = air plus ground
h = height of obstacle
i = indexing subscript
n = number engines
o = touchoff condition
opt = optimum value
SL = sea level, standard conditions
x = horizontal direction
y = vertical direction

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1. Introduction

VECTORED-THRUST, jet-propelled STOL aircraft are defined herein as those aircraft which achieve short takeoff performance by using a vertical component of the thrust to supplement the normal lifting forces (see Fig. 1). This requires some feature of the aircraft for turning the propulsion unit or at least the thrust producing component.

A great deal of work has been done in the field of STOL takeoff performance, and in all probability every large or even moderate manufacturer of high-performance aircraft has a computer program for generating the takeoff trajectory of a STOL airplane. References 1 and 2 present parametric studies of STOL aircraft takeoff performance in which such programs were utilized. An analytical investigation of takeoff performance was deemed warranted, however, since it would reveal the more significant aircraft parameters and determine their functional relationship to performance.

The total takeoff distance X_{a+g} is the sum of the ground run distance X_g and air distance X_a (see Fig. 1b). The ground distance is a straightforward calculation since usually only a single degree of freedom is involved. Most of the newer textbooks on elementary performance treat the problem adequately for CTOL aircraft.[3,4] Reference 5 presents a brief tradeoff study for estimating the advantages of operating a VTOL aircraft in the STOL mode, but omits the details of the analysis used.

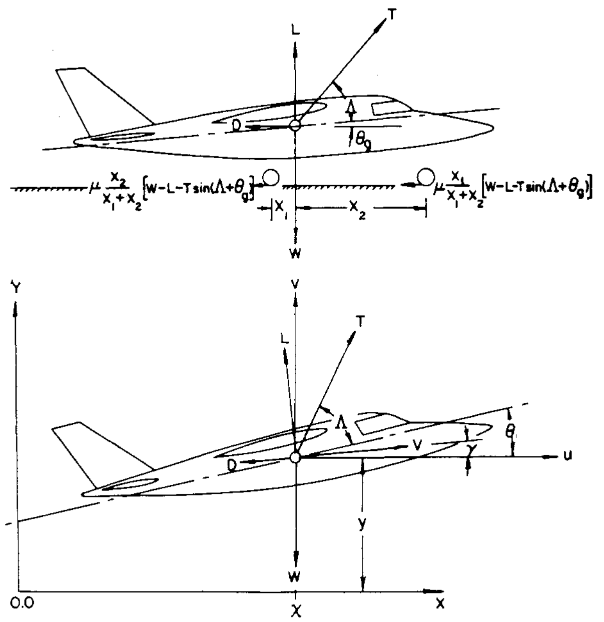

Fig. 1a Force diagrams.

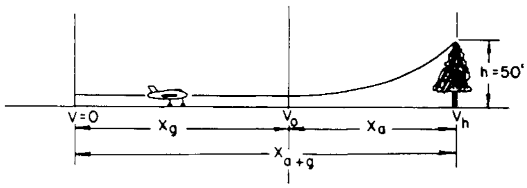

Fig. 1b Trajectory diagram.

On the other hand, the air distance calculation for CTOL aircraft is not even presented in Refs. 3 and 4. Reference 6 simply states that the air distance is "approximately 10% of the ground distance." Some aircraft companies were computing the air distance for CTOL aircraft, as late as 1960, using the specification requirement (MIL-C-5011A) that the air speed at the 50-ft obstacle (defined as V_{50} herein) should be at least 20% greater than the stalling speed of the aircraft. This, together with the requirement that the minimum touchoff speed is 110% of the poweroff stall speed, and the assumption that the horizontal acceleration is constant and equal to the acceleration at touchoff, enables one to compute the air distance simply using an energy method. The result is usually erroneous, however, since the speed at the 50-ft obstacle depends on the aircraft's physical characteristics such as the thrust-to-weight and lift-to-drag ratios, wing loading, and C_{L_c} as shown herein.

Early references dealing with the air distance calculation of CTOL aircraft assume that the airplane executes a transition of constant radius which is followed by a rectilinear flight path of constant rate of climb until the 50-ft obstacle is reached. However, it is believed that most aircraft never attain equilibrium flight by the time the 50-ft obstacle is cleared. Therefore, a dynamic problem must be solved wherein account is taken of the fact that the vertical acceleration of the aircraft increases progressively after touchoff, owing to the fact that the lift increases in proportion to the flight velocity squared. The horizontal acceleration usually decreases with increasing velocity but rarely becomes zero, the condition necessary for equilibrium flight. Reference 7, which deals with CTOL aircraft, takes this approach and is probably the best theoretical treatment of the problem. However, the analysis given therein is somewhat lengthy. A number of calculated trajectories made by the authors support this nonequilibrium hypothesis as well, and certainly the authors of Refs. 1 and 2 worked along these lines.

References 8–10 deal with the takeoff performance of CTOL airplanes, but similar studies for STOL aircraft are not presently available. It is the purpose of this report to provide a method for estimating the takeoff performance of both CTOL and STOL airplanes rapidly and to present the results of a parametric study which should prove of interest to the design engineer.

2. Derivation of Equations

2.1 Calculation of Ground Distance

The ground distance can be determined, upon letting $dV/dt = a_x = V\,dV/dx$, by

$$X_g = \int_o^{V_o} \frac{VdV}{a_x} \tag{1}$$

where $V = u$, and u and a_x are the horizontal components of velocity and acceleration, respectively. If a_x is constant or can be expressed as a linear or quadratic function of V, Eq. (1) is easily integrated, and the ground distance can be determined, presuming that V_o, the velocity at touchoff, is known.

For simplicity it is assumed that the forward acceleration can be considered a constant during the takeoff run.‡ In order to justify this assumption, an analysis was made comparing the average acceleration with an acceleration of the form: $a_x = K_1 - K_2V^2$, as used in more recent texts.[3,4] This comparative analysis has shown that, if the acceleration (or the accelerating force) changes less than 40% of the initial value by the time V_o is reached, the "exact" and constant acceleration ground distance calculations differ by only 2.3%.[12]

The forward acceleration for the general STOL airplane can be expressed (see Fig. 1):

$$a_x = \frac{T_x - D_{\text{av}} - \mu[W - T_y - L_{\text{av}}]}{W/g} \tag{2}$$

where L_{av} and D_{av} are the average lift and drag values pertaining to the ground run and are evaluated at $V = 0.707V_o$. The horizontal and vertical components of the thrust, T_x and T_y, are also average values. The thrust components are calculated for the general, multiengine STOL aircraft as

$$T_x = \sum_{i=1}^{n} (T_{\text{av}})_i \cos(\Lambda_i + \theta_g) \tag{3}$$

$$T_y = \sum_{i=1}^{n} (T_{\text{av}})_i \sin(\Lambda_i + \theta_g) \tag{4}$$

Equations (3) and (4) are written with the assumption that the thrust and inclination angle are constant during the ground run, i.e., there are no provisions in the present analysis for the case where the thrust magnitude or inclination are programed as functions of time, velocity, or distance, during the ground run.

Upon substitution of Eq. (2) into (1) and integrating, the ground run for a vectored-thrust STOL aircraft can be expressed as

$$X_g = \frac{V_o^2}{2g} \Bigg/ \left\{ \frac{T_x}{W} - \mu\left[1 - \frac{T_y}{w}\right] - \frac{1}{2}\,[C_{D_g} - \mu C_{L_g}]\,\frac{(\rho/2)V_o^2}{W/S} \right\} \tag{5}$$

The velocity at touchoff can be either the minimum poweron flying speed $V_{o_{\min}}$ or some velocity in excess of $V_{o_{\min}}$. Supposing that the aircraft is flown in a manner such that vertical equilibrium is established at touchoff, the touchoff velocity

‡ This approximation, attributed to Hartman[11] is used in most of the earlier reference texts.

becomes related to the lift coefficient as

$$W = T_{y_o} + L_o = T_{y_o} + C_{L_o}S(\rho/2)V_o^2 \tag{6}$$

where

$$T_{y_o} = \sum_{i=1}^{n} (T_0)_i \sin(\Lambda_i + \theta_g) \tag{7}$$

The touchoff velocity can be determined from Eq. (6):

$$V_o = \{(2/\rho_{SL})[(W/S)(1/\sigma C_{L_o})] [1 - (T_{y_o}/W)]\}^{1/2} \tag{8}$$

and substituted into the denominator of Eq. (5). If the assumption is made that the vertical component of thrust is nearly independent of velocity,

$$1 - (T_y/W) \cong 1 - (T_{y_o}/W)$$

Equation (5) can be simplified:

$$X_g = \frac{V_o^2}{2g} \Bigg/ \left\{ \frac{T_x}{W} - \left[1 - \frac{T_y}{W}\right] \times \left[\mu + \frac{1}{2}\left(\frac{C_{D_g} - \mu C_{L_g}}{C_{L_o}}\right)\right]\right\} \tag{9}$$

The reader is cautioned that Eq. (9) applies only to those cases where vertical equilibrium is established at touchoff, i.e., where V_o corresponds to $V_{o_{\min}}$. If an initial vertical acceleration is induced upon "rocking" the aircraft at touchoff, for the case where $V_o > V_{o_{\min}}$ and C_{L_o} is the maximum usable lift coefficient, then Eq. (5) must be used instead. Note that if $T_y/W = 0$, which represents the conventional airplane, Eqs. (5) and (9) can be shown to be identical to the ground distance equations used to calculate the takeoff performance of conventional aircraft.[6]

The form of Eq. (9) is useful for cases where the touchoff velocity occurs explicitly in the problem, and so graphs will be presented for determining ground distance in terms of the parameters that appear in this equation. It is convenient to define the denominator of Eq. (9) as the "effective forward thrust-to-weight ratio," $(T_x/W)_{\text{eff}}$, which is, of course, equal to the average forward acceleration of the airplane, expressed in g's, during the takeoff ground roll.

A plot of $(1 - T_y/W)$ vs T_x/W is presented in Fig. 2 for the evaluation of these quantities given the thrust-to-weight ratio (T/W) and thrust inclination angle $(\Lambda + \theta_g)$ (applies to the equal engine inclination case only). Figure 3 presents the velocity at touchoff as a function of effective vertical wing loading, where $(W - T_{y_o})/S = (W/S)(1 - T_{y_o}/W)$, for various lift coefficient at touchoff. This figure is the graphical representation of Eq. (8) and assumes that the airplane

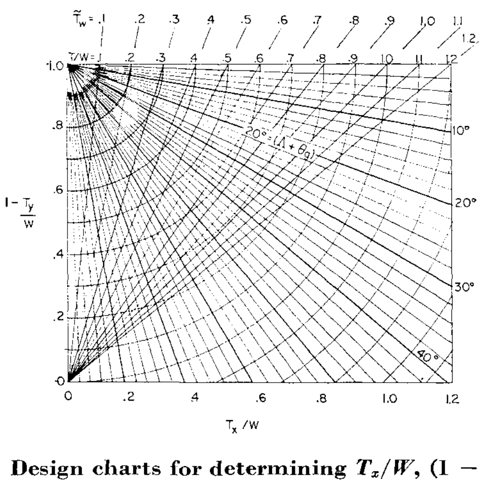

Fig. 2 Design charts for determining T_x/W, $(1 - T_y/W)$, $\tilde{T}_w$ given T/W and $(\Lambda + \theta_g)$. $T_x/W = (T/W)\cos(\Lambda + \theta_g)$ $T_y/W = (1/W)\sin(\Lambda + \theta_g)$. Note: constant T/W values are circular arcs, and plot applies to equal inclination case only.

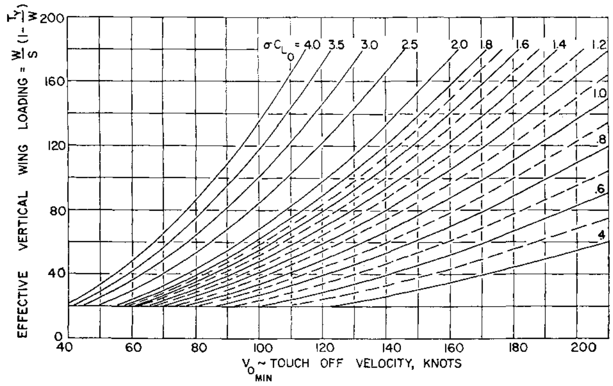

Fig. 3 Variation of touchoff velocity with effective vertical wing loading for various touchoff lift coefficients.

is in vertical equilibrium at touchoff ($a_{y_o} = 0$). Figure 4 presents the takeoff ground roll distance X_g, as a function of V_g for various effective forward thrust-to-weight ratios $(T_x/W)_{\text{eff}}$. Figures 2–4 serve as design charts for rapidly estimating the ground roll distance of conventional or STOL airplanes.

An alternative method of treating the takeoff distance calculation is to eliminate V_o, as may be done by substituting Eq. (8) into (9), and dividing the right-hand side, numerator and denominator, by $(1 - T_y/W)$. The ground distance is then given by

$$X_g = \frac{\left(\dfrac{1}{\rho_{SL}g}\right)\left(\dfrac{W}{S}\dfrac{1}{\sigma C_{L_o}}\right)}{\dfrac{T_x/W}{1 - T_y/W} - \left[\mu + \dfrac{1}{2}\dfrac{C_{L_g}}{C_{L_o}}\left(\dfrac{1}{(L/D)_g} - \mu\right)\right]} \tag{10}$$

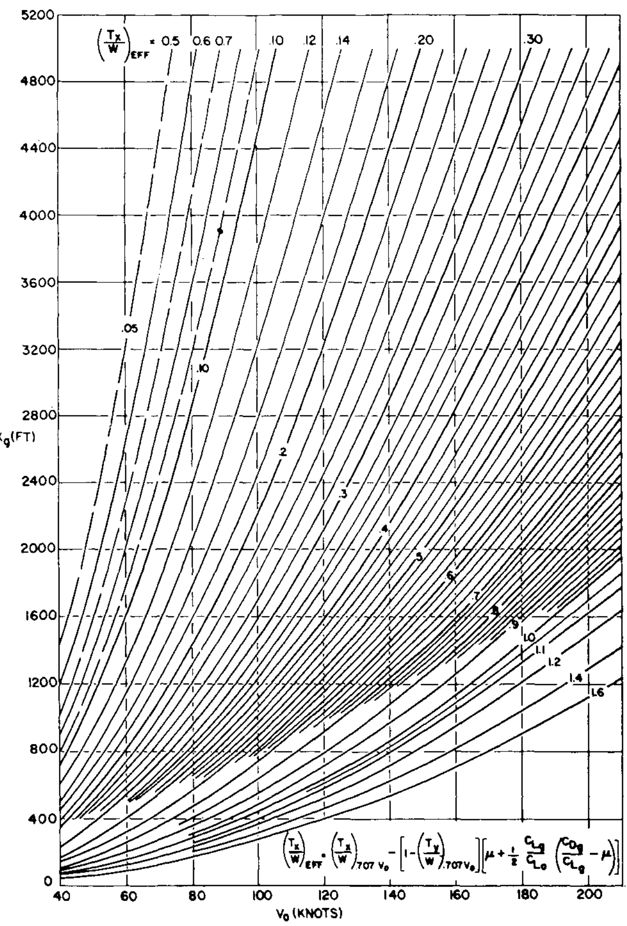

Fig. 4 Takeoff ground distance as a function of touchoff velocity ~ STOL or conventional aircraft.

This equation serves to introduce two new parameters, the quantities $(W/S)\ (1/\sigma C_{L_o})$ which, in a sense, is an effective wing loading, and $(T_x/W)/(1 - T_y/W)$ which is an STOL thrust-to-weight parameter.

It may be shown by differentiation of Eq. (9),[5] that for a given (T/W) ratio the STOL takeoff ground run will be a minimum when the engines have equal thrust inclination, where the inclination is given by

$$\sin(\Lambda + \theta_g) = T/W \tag{11}$$

Figure 2 indicates that constant values of $T_x/W(1 - T_y/W)^{-1}$ (abbreviated $\bar{T}_w$ in Fig. 2) are radial lines emanating from the origin. The optimum value of $(\Lambda + \theta_g)$ may be determined by the condition of tangency of the radial line with the circular arcs $(T/W) = $ const.

2.2 Calculation of Air Distance

2.2.1 *General expressions*

The air distance calculation is somewhat more complex than the foregoing analysis since the aircraft now has 2 degrees of freedom, and hence the motion is described by two simultaneous differential equations. These equations of motion can be written using Cartesian axes (see Fig. 1) as follows:

$$a_x = \frac{du}{dt} = \frac{T_x - L_c \sin\gamma - D_c \cos\gamma}{W/g} \tag{12}$$

$$a_y = \frac{dv}{dt} = \frac{T_y + L_c \cos\gamma - D_c \sin\gamma - W}{W/g} \tag{13}$$

where u and v are the horizontal and vertical velocity components, respectively, and, during climbing flight

$$T_x = \sum_{i=1}^{n} T_i \cos(\Lambda_i + \alpha_c + \gamma) \tag{14}$$

$$T_y = \sum_{i=1}^{n} T_i \sin(\Lambda_i + \alpha_c + \gamma) \tag{15}$$

Consideration of the flight-path geometry yields the following additional relations:

$$\tan\gamma = v/u \tag{16}$$

$$V = (u^2 + v^2)^{1/2} \tag{17}$$

The lift and drag and the coefficients C_{LC} and C_{DC} can have an angle-of-attack dependence during climbing flight. Remembering that, in general, the engine thrust T_i is dependent on velocity, it is seen, from Eqs. (12) and (13), that the horizontal and vertical acceleration are functions of the aircraft's velocity, flight-path angle, and angle of attack.

The system of equations, Eqs. (12–17), can be integrated numerically to determine the trajectory of the airplane if C_{LC}, C_{DC}, and T_i are completely specified. An analytical solution is more desirable, however, since the key parameters are usually indicated and a more organized presentation of results achieved. The reader is referred to Ref. 12 for a more detailed derivation of the analysis which follows.

A great deal of simplification results upon the assumption of constant horizontal acceleration. Further assumptions are: 1) aircraft weight and thrust inclination Λ_i are constant; 2) angle of attack, and hence C_{LC} and C_{DC}, are held constant; and 3) angle of climb is small so that $\cos\gamma \cong 1$ and $\sin\gamma \cong 0$, and hence $(\Lambda_i + \alpha_c + \gamma) \cong (\Lambda_i + \alpha_c)$, which is perhaps the most limiting assumption. Attention is drawn to the fact that while the horizontal acceleration is assumed constant, its magnitude depends on the speed range under consideration, namely V_o to V_h. Since V_h is unknown, the horizontal acceleration is therefore unknown also.

The horizontal acceleration $a_{x_{av}}$ will correspond to the average velocity squared:

$$V_{av}^2 = (V_h^2 + V_o^2)/2 = [(V_h/V_o)^2 + 1](V_o^2/2) \tag{18}$$

Therefore, from Eq. (12)

$$\frac{a_{x_{av}}}{g} = \left(\frac{T_x}{W}\right)_{av} - \frac{1}{2}\frac{C_{DC}}{C_{LC}}\frac{C_{LC}}{C_{L_o}}\left(1 - \frac{T_{y_o}}{W}\right)\left(\frac{V_h^2}{V_o^2} + 1\right) \tag{19}$$

For initial evaluation $T_{x_{av}}$ can be approximated by using T_{x_o} which is known. Equation (19) can be expressed as a linear function of V_h/V_o, for $1. < V_h/V_o < 1.5$, which expedites calculations:

$$\frac{a_{x_{av}}}{g} = \left(\frac{T_x}{W}\right)_{av} - \frac{C_{DC}}{C_{LC}}\frac{C_{LC}}{C_{L_o}}\left(1 - \frac{T_{y_o}}{W}\right)\frac{V_h}{V_o} \tag{20}$$

The vertical distance is calculated:

$$\int_0^h dy = \int_{t_o}^{t_h} v dt = \frac{1}{a_{x_{av}}}\int_{V_o}^{V} v dV \tag{21}$$

where

$$v = \frac{1}{a_{x_{av}}}\int_{V_o}^{V} (K_y + K_L V^2) dV \tag{22}$$

and

$$K_y = -g[1 - (T_{y_o}/W)] \tag{23}$$

$$K_L = (\rho_{SL} g/2) C_{LC}/C_{L_o}[(W/S)(1/\sigma C_{L_o})]^{-1} \tag{24}$$

Upon integration of Eqs. (21) and (22), substitution of Eqs. (23) and (24), and rearrangement, the average forward acceleration as a function of V_h/V_o and C_{LC}/C_{L_o} (a known constant) becomes

$$\frac{V_g}{V_o}\left(1 - \frac{T_{y_o}}{W}\right)^{-1/2}\frac{a_{x_{av}}}{g} = \left(\frac{V_h}{V_o} - 1\right) \times \left\{\left(\frac{C_{LC}}{C_{L_o}} - 1\right) + \frac{C_{LC}}{C_{L_o}}\left[\frac{2}{3}\left(\frac{V_h}{V_o} - 1\right) + \frac{1}{6}\left(\frac{V_h}{V_o} - 1\right)^2\right]\right\}^{1/2} \tag{25}$$

where $V_g = (2\,gh)^{1/2}$. Equations (20) and (25) provide a set of simultaneous equations for determining $(a_{x_{av}}/g)$ and (V_h/V_o). A graphical solution is used to solve these equations, and hence both sides of Eq. (20) must be multiplied by $(V_g/V_o)[1 - (T_{y_o}/W)]^{-1/2}$. The parameter $(V_g/V_o)(1 - T_{y_o}/W)^{-1/2}(a_{x_{av}}/g)$ is defined as Z for brevity.

Figure 5 presents curves of Z vs V_h/V_o for various values of C_{LC}/C_{L_o} based on Eq. (25). This figure is general in nature and applies to all types of aircraft whose thrust varies only slightly with velocity. Superposition of the straight line corresponding to Eq. (20) following multiplication by $(V_g/V_o)(1 - T_{y_o}/W)^{-1/2}$ produces the solution values Z and V_h/V_o, as illustrated in Fig. 5. For those aircraft whose thrust varies significantly with velocity, a second iteration can be made using a value for $T_{x_{av}}$ which is based on the value of V_h obtained from the first approximation.

The value of Z, obtained from Eq. (20) modified by the substitution of Eq. (8) for V_o and introduction of the effective wing loading and STOL thrust-to-weight parameters, becomes

$$Z = \frac{(\rho_{SL} g h)^{1/2}}{[(W/S)(1/\sigma C_{L_o})]^{1/2}}\left[\frac{T_x/W}{1 - (T_y/W)} - \frac{C_{DC}}{C_{LC}}\frac{C_{LC}}{C_{L_o}}\frac{V_h}{V_o}\right] \tag{26}$$

The air distance X_a is calculated directly from Eq. (1), upon modification of the limits of integration, to yield

$$X_a = (V_o^2/2a_{x_{av}})[(V_h/V_o)^2 - 1] \tag{27}$$

which may be expressed as

$$X_a = h \frac{[(W/S)(1/\sigma C_{L_o})]^{1/2}}{(\rho_{SL} g h)^{1/2}} \frac{(V_h/V_o)^2 - 1}{Z} \qquad (28)$$

The graphical solution given previously for determining $a_{x_{av}}$ and V_h and hence X_a produces a reasonably accurate result, consistent with the ease of executing the calculation.

Reference 12 indicates that if one is willing to relax the requirements for accuracy, a further simplification permits the elimination of the graphical solution. Upon curve fitting and algebraic manipulation, the air distance is calculated:

$$X_a \cong \frac{3.08h}{[\rho_{SL} g h]^{0.627}} \left(\frac{W}{S}\frac{1}{\sigma C_{L_o}}\right)^{0.627} \left[\frac{T_x/W}{1 - T_y/W} - \frac{C_D}{C_L}\right]^{-0.253} \qquad (29)$$

Use of this equation is restricted to values of the last bracket greater than 0.05. Equation (29) is more important for its functional form than for generating numerical results, as will be shown subsequently.

Equation (29) indicates that the minimum air distance will occur when $(T_x/W)/(1 - T_y/W)$ is a maximum, which is exactly the condition requiring that the ground distance be a minimum. It is therefore concluded that both the ground and air distance, and hence their sum, are optimized by the thrust inclination:

$$\Lambda_{opt} = \sin^{-1}(T/W) \qquad (\theta_g = 0)$$

for the case where Λ is constant.

3. Discussion of Results

3.1 Parametric Study

Two operational methods of taking-off an STOL aircraft are described in Refs. 1 and 2, and these are:

Method a—to use a single inclination of the thrust vector (relative to the aircraft) during the ground roll and climbing portions of the takeoff, and the inclination chosen would correspond to a minimum total distance, $\Lambda = (\Lambda_{opt})_{a+g}$, and

Method b—to point the thrust vector in a horizontal direction during the ground roll ($\Lambda = 0$)§ and then to rotate the thrust vector instantaneously to the optimum angle for climbing flight, $\Lambda = (\Lambda_{opt})_a$.

It is realized, of course, that the air distance is the same for methods a and b. The ground distance is shorter for method b because the horizontal acceleration is higher in this case, whereas the touchoff speed is the same in both methods. The method b takeoff is more difficult to achieve practically and for this reason, and reasons of brevity, is not considered further herein. Takeoff performance of STOL aircraft using this method is presented in Ref. 12.

The ground, air ($h = 50$ ft), and total takeoff distances were computed parametrically using the graphical analysis for air distance for over 100 hypothetical conventional and STOL aircraft, where the latter were operated according to method a. The primary parameters considered were $(W/S)(1/\sigma C_{L_o})$ and thrust-to-weight ratio (T/W). For simplicity it was assumed that the thrust was independent of airspeed and that the lift-to-drag ratio was 9.0. An additional study was made for STOL aircraft in which the L/D ratio was varied from 3 to ∞.

It was shown by Perkins and Hage[6] that the total takeoff distance of CTOL aircraft could be correlated on the basis of a single parameter, namely, $(W/S)\ (1/\sigma C_{L_o})(W/T)$. If one examines the ground distance for STOL aircraft, as shown by Eq. (10), it is seen that there are four primary parameters: $(W/S)(1/\sigma C_{L_o})$, T/W, Λ, and $(L/D)_g$ affecting STOL takeoff performance, whereas θ_g, C_{L_g}/C_{L_o}, and μ can be considered as secondary parameters. For the purpose of this

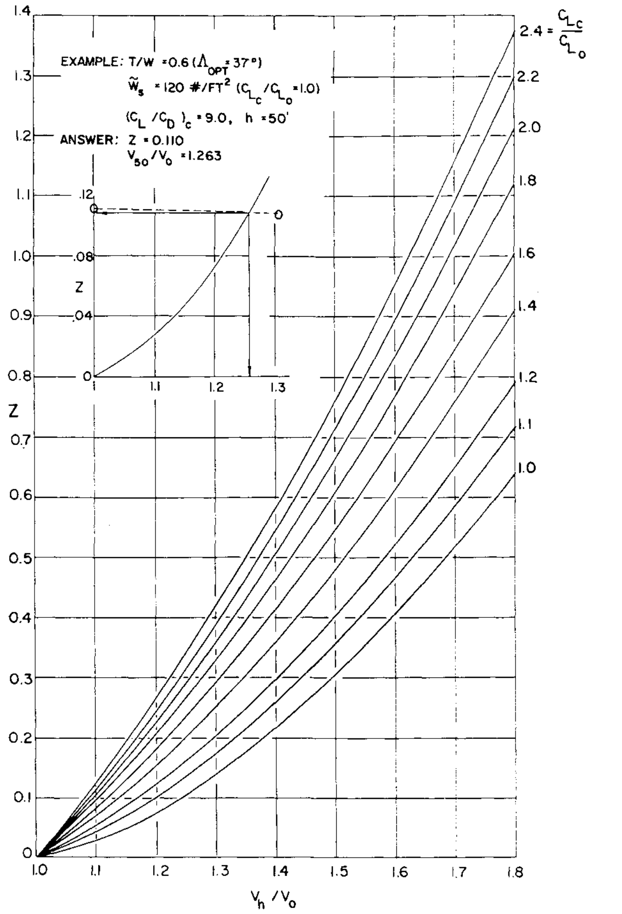

Fig. 5 Air distance calculation design chart for determining Z and V_h/V_o.

report, values of 0°, 1, and 0.03, respectively, were chosen for these secondary parameters. A further simplification can be made if one assumes that Λ is taken to be its optimum value so that the parameters T/W and Λ are replaced by the STOL T/W parameter: $[(T_x/W)(1 - T_y/W)]_{opt}$. Conventional aircraft are characterized by the fact that the STOL thrust-to-weight ratio parameter is replaced by T/W.

It is found,[12] that the calculated ground distance for both CTOL and STOL aircraft can be shown to be a function of a single parameter, namely,

$$(W/S)(1/\sigma C_{L_o})[(T_x/W)/(1 - T_y/W)]^{-1}$$

which is in keeping with the result shown earlier by Perkins and Hage for CTOL aircraft. Nonoptimum values of thrust inclination are also correlated by this parameter. The value of lift-to-drag ratio does not appear explicitly and this is permissible for L/D ratios greater than 8. For L/D less than 8, the ground distance becomes more dependent on the L/D ratio, as discussed below.

The air distance calculations indicated that it is necessary to use a parameter of the form

$$[(W/S)(1/\sigma C_{L_o})]^{0.627}\{[(T_x/W)/(1 - T_y/W)] - (C_D/C_L)\}^{-0.253}$$

in order to correlate the various results, see Eq. (29). Figure 6 presents results obtained from these studies and indicates that the parameter chosen for the correlation should be suitable for preliminary design purposes. For more exact results the graphical method described earlier should be used.

The total takeoff distance (sum of the ground and air distance) is presented in Fig. 7 as a function of $(W/S)(1/\sigma C_{L_o})[(T_x/W)/(1 - T_y/W)]^{-1}$, which shows that the total distance is reasonably well correlated by the same parameter

§ Assuming, for simplicity, that θ_g is zero, see Fig. 1.

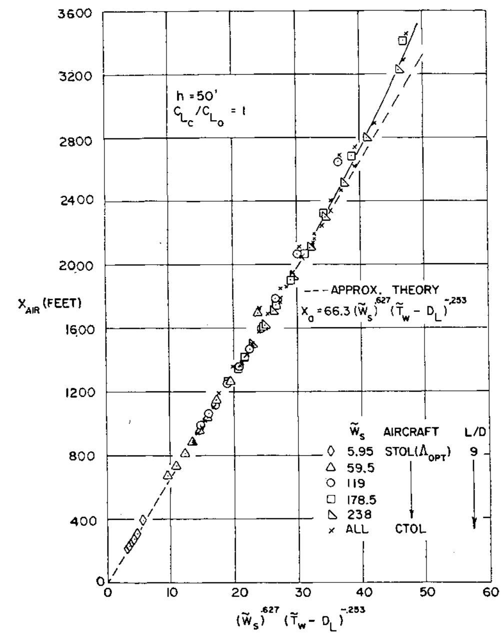

Fig. 6 Air distance as a function of $\tilde{W}_s$ and $\tilde{T}_w$ for CTOL and STOL aircraft.

as used for the ground distance. This applies for both CTOL and STOL aircraft, $L/D = 9.0$, where both optimum and nonoptimum values of thrust inclination were used in the STOL cases. The results of Ref. 6 are shown for comparison purposes, and reasonably good agreement is shown.

It is emphasized that the preceding results correspond to $L/D = 9.0$, and that the simple result achieved for these cases will not follow for the lower L/D values. Figure 8 presents a correlation of the STOL cases for L/D ratios from 3 to 12, and in this case the correlation parameter is closer to the form used for air distance as given previously except that the values of the exponents change to 0.86 and -0.67, respectively. This parameter is more general than that used in Fig. 7 and should really supersede it. Because of the simplicity of the form of correlating parameter used in Fig. 7, it should, however, be retained for aircraft with high lift-to-drag ratios, i.e., $L/D \geq 8$.

3.2 Takeoff Performance of CTOL and STOL Aircraft

In order to facilitate a more direct comparison of STOL and CTOL takeoff performance, curves of ground, air, and total

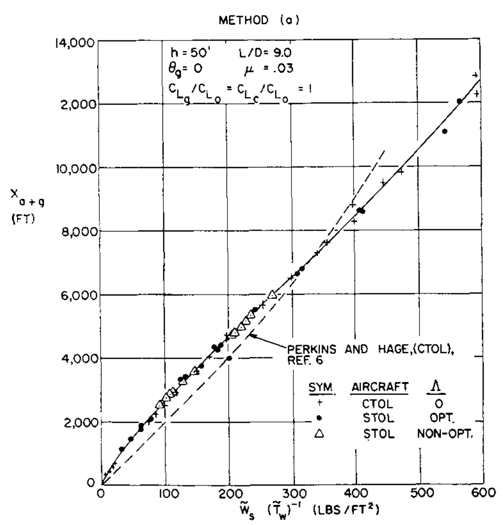

Fig. 7 Total distance vs $\tilde{W}_s\ (\tilde{T}_w)^{-1}$ for CTOL and STOL aircraft.

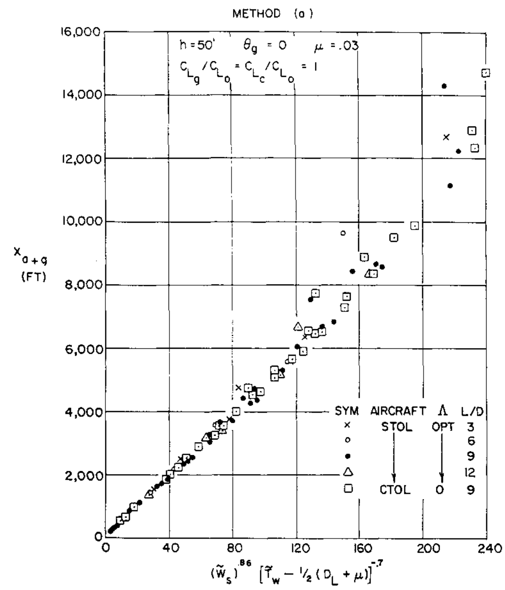

Fig. 8 Total distance vs $(\tilde{W}_s)$ 0.86 $[\tilde{T}_w - \frac{1}{2}(D_L + \mu)]^{-0.7}$ for CTOL and STOL aircraft.

distance as a function of T/W ratio are presented in Figs. 9–12, for various effective wing loadings.¶ The curves presented in these figures permit a more rapid estimate of the tradeoff of wing loading and thrust vectoring for a required takeoff distance. The STOL aircraft assumed in these figures utilized the optimum thrust inclination, as noted.

Figure 9 presents a comparison of the ground distance required for takeoff for CTOL and STOL aircraft, where the latter are operated according to method a. Again it is noted that the ground distance saved by thrust vectoring is generally insignificant for T/W ratios less than 0.4. As the T/W ratio increases above this value, the STOL and CTOL curves separate appreciably. On the other hand, the ground distance of both are reduced significantly, so that the potential saving made possible by thrust vectoring becomes less inviting. It is indicated that really significant savings are reserved for those aircraft whose effective wing loadings are very high. However, the effective wing loadings can be reduced with high-lift devices, and these appear to offer an attractive alternative to thrust vectoring.

Figure 10 presents a comparison of the CTOL and STOL air distances as a function of T/W ratio for various effective

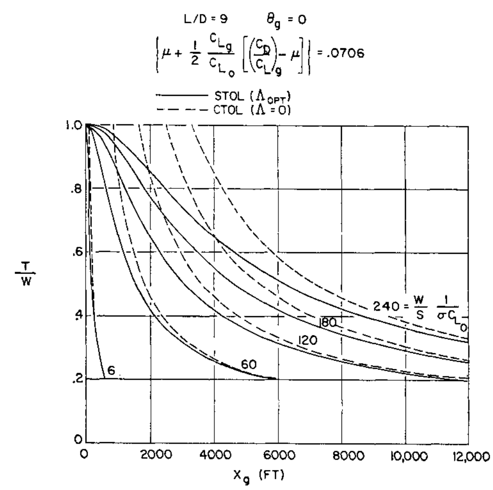

Fig. 9 Comparison of STOL and CTOL ground distances —method a.

¶ The actual values of $(W/S)\ (1/\sigma C_{L_o})$ used are 0.99 times the value shown on the graphs, which are "rounded-off" numbers.

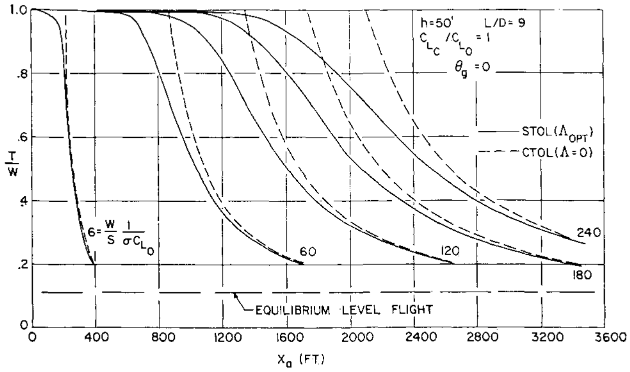

Fig. 10 Comparison of STOL and CTOL air distances.

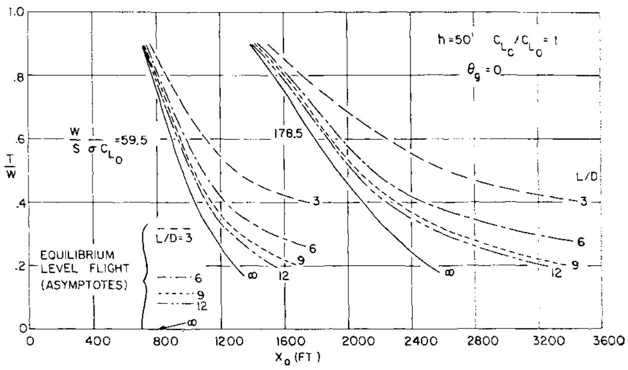

Fig. 11 Effect of L/D on STOL air distance.

wing loadings. It is observed that the air distances approach infinity asymptotically as equilibrium level flight is approached, i.e.,

$$[(T_x/W)/(1 - T_y/W)] \rightarrow C_D/C_L$$

The corresponding value of T/W is equal to 0.1103 for these cases where $\Lambda = \Lambda_{opt}$ and $L/D = 9.0$. Figure 10 also indicates that the reduction in air distance achieved by thrust vectoring is very small below $T/W = 0.7$; in fact, the maximum saving is about 200 ft which occurs at the highest wing loading for which calculations were made. Obviously the advantages of thrust vectoring are realized in the ground distance portion of takeoff.

The effect of L/D ratio on air distance is shown in Fig. 11, which presents curves of X_a vs T/W for various L/D ratios, for two different effective wing loadings. The equilibrium level flight values are indicated in this figure for the different L/D ratios. It is seen that L/D ratio has a significant effect on the air distance, and that the designer has to be careful that he does not negate the gains he might claim by thrust vectoring with a poor aerodynamic configuration.

Figure 12 presents a comparison of CTOL and STOL takeoff distances. Figures of this type provide the STOL aircraft designer a rapid means of studying the tradeoffs of thrust vectoring, wing loading, and maximum lift coefficient. The particular results presented in this figure are limited mainly by the specific L/D ratio chosen, namely $L/D = 9.0$. As an example of the use of this type of presentation, given the field length from which the aircraft is to operate, the designer draws a vertical line at the specified value of X_{a+g} and then cross plots the values of T/W vs $(W/S)(1/\sigma C_{L_o})$ for STOL and CTOL aircraft which meet the field requirements. The determination of specific values of T/W or wing loading then must be made on the basis of other design considerations, such as range, landing, cruising performance, etc. The main conclusions to be drawn from these design curves are:

1) Thrust vectoring is highly beneficial for reducing ground and total takeoff distances, but only at the high T/W ratios, and high effective wing loadings. The performance gains made by vectoring can be achieved in other ways, such as by reducing wing loadings or utilizing high-lift devices.

2) The potential gains owing to thrust vectoring can be negated by reduced L/D ratios, so that it is necessary to retain a reasonably high degree of "aerodynamic cleanliness" in STOL designs.

4. Conclusions

The methods developed herein for the calculation of ground and air takeoff distances have been derived for the general vectored-thrust jet-propelled STOL aircraft. It has been shown throughout the report that CTOL aircraft can be treated as a special case of STOL aircraft, and calculated results using these methods for CTOL aircraft were found to be consistent with the flight-test data correlation presented in Ref. 6. These methods have the particular advantage of simplicity associated with the assumption of constant average horizontal accelerations. There is, however, the restriction that small flight-path angles were assumed in the air distance solution, which limits the application of that method to STOL aircraft having thrust-to-weight ratios less than 0.9 and effective wing loadings greater than about 30#/ft². This method is shown to be applicable to aircraft whose thrust magnitude has a moderate velocity dependence by using an iteration technique.

Parametric studies of CTOL and STOL takeoff performance were conducted, and it was found that this performance could be correlated on the basis of an STOL effective thrust-to-weight ratio $(T_x/W)/(1 - T_y/W)$, an effective wing loading $(W/S)(1/\sigma C_{L_o})$, and the usual L/D ratio. These studies should prove useful for preliminary design purposes, whereas more exact values can be obtained from the analytical/graphical analysis provided.

Numerical calculations of takeoff performance have produced results of apparent practical value, but the lack of flight-test data for vectored-thrust STOL aircraft precludes a direct comparison of these results with test data. Hopefully, as operational experience with STOL aircraft increases, the analytical predictions made herein will prove valid. These numerical computations have indicated that:

1) If STOL aircraft, operating at optimum thrust inclination, are to show significant reductions in total takoff distances when compared with their conventional counterparts, then thrust-to-weight ratios in excess of 0.6 are required. The reductions then effected are shown to be strongly related to the effective wing loadings. Not much advantage is shown for those effective wing loadings less than 60#/ft².

2) Improvements in performance equivalent to those gained by thrust vectoring can be achieved through the combined effects of increased lift capabilities and lower wing

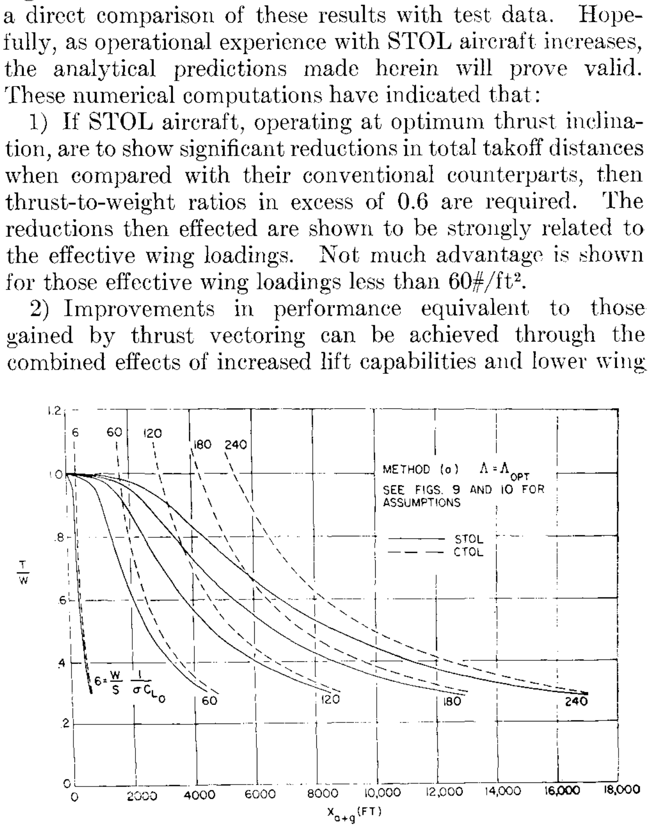

Fig. 12 Comparison of STOL and CTOL total takeoff distances (h = 50 ft), method a.

loadings, and that developments in this direction are equally important.

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Comparative Projections of Low-Disk-Loading VTOL Aircraft for Civil Applications

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The technical and economic characteristics of three types of low-disk-loading VTOL aircraft for short-haul civil transport in 1985 are forecast. Comparisons of the types indicate that all three will find applications in the civil market of the future and that selection of one type in preference to another depends principally on requirements of aircraft size and operating stage length. In all size and payload categories, the helicopter will have the lowest cost per seat-mile for short stage lengths (10 to 25 naut miles). In larger transport categories (30 to 60 passengers), the compound helicopter will become cost-effective relative to the helicopter when it is designed for a cruise speed 50% greater than that of the helicopter. The composite VTOL, represented in this study by the tilting-proprotor aircraft, is the most economical in the larger transport categories at stage lengths greater than 30 naut miles. This kind of aircraft, designed for a 400-knot cruise speed, will provide 100- to 300-mile intercity service at a fare-cost comparable to that of advanced-technology jet "airbuses," while operating either from airports or from VTOL terminals.

Introduction

THE possibilities for applications of VTOL aircraft in the urban transportation systems of the future are immense. They may range from the multitude of specialized tasks of today's small helicopter, through passenger and utility transport within a megalopolis, to intercity transport between city centers in direct competition with airplanes, buses, trains and other future modes of transportation.

In exploring this spectrum, four general classes of aircraft have been considered: executive class with payload capability up to 1200 lb (6 places), utility class with payload up to 2500 lb (12 places), light transport with 6000 lb (28 passengers), and medium transport with payload capability up to 12,000 lb (58 passengers). Helicopters, compound helicopters, and tilting-proprotor aircraft were developed for each class and perturbations of design cruise speed and design cruise range were examined for each. The independent parameters of the aircraft configurations that were studied are given in Table 1.

Design characteristics of these aircraft—such as gross weight, weight empty, fuel required, and installed power—were derived from technical characteristics that include the effects of predicted advances in technology for the 1975–1985 time period. The technical characteristics which have a significant impact on the results of the study are the lift-drag ratio, hover power loading, weight-empty-to-gross-weight ratio, and specific fuel consumption.

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